DS-GA 3001 005 | Lecture 5

Reinforcement Learning

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DS-GA 3001 RL Curriculum

Reinforcement Learning:

- Introduction to Reinforcement Learning
- Multi-armed Bandit
- Dynamic Programming on Markov Decision Process
- Model-free Reinforcement Learning
- **Value Function Approximation (Deep RL)**
- Policy Function Approximation (Actor-Critic)
- Planning from a Model of the Environment
- Examples of Industrial Applications
- Advanced Topics and Development Platforms

Reinforcement Learning

Last week: Model Free RL

- Monte Carlo and Temporal Difference
- Sample based Prediction and Control
- Off policy Learning

Today: Value Function Approximation

- **Categories of Functions**
- **Approximation of State Update and Value Functions**
- **Deep Reinforcement Learning**

Categories of Functions in Reinforcement Learning

RL Functions and Approximations

- **All key components of an RL agent are functions**
  - **State update functions** map observations to states
  - **Value functions** map states to values
  - **Policies** map states to actions
  - **Models** map states and actions to next states and rewards
- **Functions can be parameterized and approximated** by linear or non-linear gradient methods e.g., Deep Learning
- **If approximated, this is called RL with function approximation**
- **Challenge:** Supervised learning assumptions are often violated
- **Deep reinforcement learning** has recently led to breakthoughs in AI and is an active field of research

Why Approximate?

Tabular methods do not scale to large state spaces

- **So far we defined value functions as lookup tables**, where every state s has an entry $v(s)$, or set of $q(s, a)$
- **Many RL problems contain a very large number of states:**
  - **Chess**: 10^{120} states
  - **Go**: 10^{170} states
  - **Helicopter**: Continuous state space
  - **Robot**: Infinite state space (physical universe)
- **Problem with large MDPs:**
  - There are too many states/actions to store in memory
  - Learning the value of each state individually is too slow
  - Individual states are often not fully observable
  - Exact nature of a state may never happen again

Types of Function Approximation

Any function approximator can be used:

- Tabular function (look-up tables)
- State aggregation (feature engineering)
- Linear function (regression, nearest-neighbors, etc)
- Non-linear function (neural network, decision tree, etc)

But RL has specific properties:

- Experience is not i.i.d. — successive time-steps are correlated
- Agent's policy affects the data it observes
- Reward and values may continuously evolve (non-stationary)
- Feedback is delayed, not instantaneous

Approximation of State Update Functions

Generalization through state space

Function to update state and engineer state features

$$s_{t+1} = \phi(s_t, o_{t+1})$$

- Represent state by a feature vector. For example:
  - GPS location and sonar readings (distance to objects) of a robot
  - Recent trends in the stock market
  - Global configuration of the board in Chess
- If the environment state is not fully observable, there is at least an implicit mapping $o_t \mapsto s_t$

Generalization through state space

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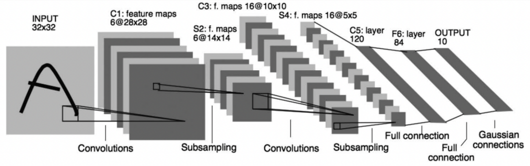
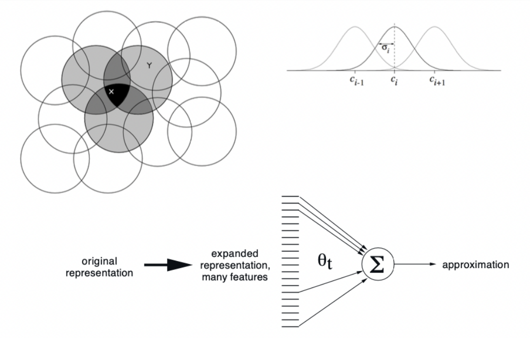


Example of State Feature Engineering

Aggregate multiple states by coarse coding:



- Generalise from seen states to unseen states
- Resulting features not Markovian $\Rightarrow$ **Partially Observable MDP**

Approximation of Value Prediction Functions

Linear Value Function Approximation

Approximate Values using a Parametric Function

$$v_w(s) \approx v_\pi(s)$$

- $v_w(s)$ can be a linear combination of state features:

$$v_w(s) \;=\; w^T \phi(s) \;=\; \sum_{i=1}^{n} w_i \, \phi_i(s)$$

- Parameters w_i can be updated incrementally by optimizing an objective measure of performance:

$$w \sim \arg\min_{w} \mathop{\mathbb{E}}_{\pi}[(v_\pi(s) - v_w(s))^2]$$

- Gradient descent can converge to global optimum if v_π known

Linear Value Function Approximation

The special case of tabular value function

- Define $v_w(s)$ as a linear combination of state features:

$$v_w(s) \; = \; w^T \phi(s) \; = \; \sum_{i=1}^{n} w_i \, \phi_i(s) \; = \; \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}^T \begin{bmatrix} \phi_1(s) \\ \vdots \\ \phi_n(s) \end{bmatrix}$$

  where n is the number of features

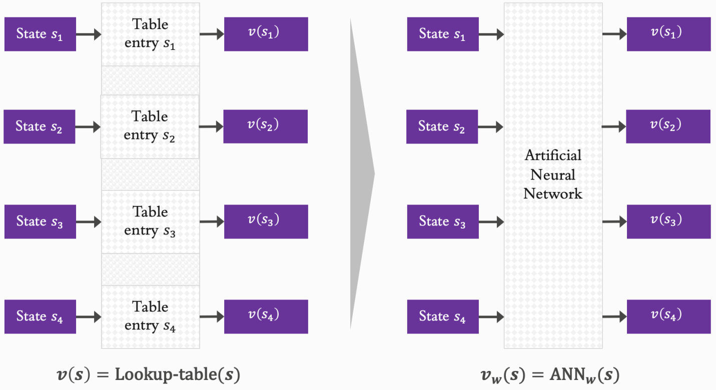
- Tabular value function (lookup table) is the special case where $n =$ number of states and where w_i is the value of each state s_i:

$$v_w(s) = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}^T \begin{bmatrix} 1(s = s_1) \\ \vdots \\ 1(s = s_n) \end{bmatrix}$$

Non-linear Value Function Approximation

Approximate Values using a Deep Neural Network

- Deep RL: Replace the look-up table by an ANN



Function Optimization by Gradient Descent

Approximate Values by Stochastic Gradient Descent

- **Goal:** Find **w** that minimizes difference between $v_w(s)$ and $v_\pi(s)$:

$$J(w) = \mathbb{E}_\pi[(v_\pi(s) - v_w(s))^2]$$

- To find a minimum of $J(w)$, define its gradient $\nabla_w J(w)$ and move w_i in the direction of negative gradient at every step:

$$\nabla_w J(w) = \begin{bmatrix} \frac{\partial J(w)}{\partial w_1} \\ \vdots \\ \frac{\partial J(w)}{\partial w_n} \end{bmatrix}$$

$$w_{t+1} = w_t - \frac{1}{2}\alpha \nabla_w \left(v_\pi(s_t) - v_w(s_t)\right)^2$$

$$w_{t+1} = w_t - \alpha \left(v_\pi(s_t) - v_w(s_t)\right) \nabla_w v_w(s_t)$$

Incremental Prediction RL Algorithms

Replace the true value of $v_\pi(s)$ by sampled target

- **In practice we do not know the true value $v_\pi(s)$**, we replace it by a target sampled using Monte-Carlo or Temporal Difference:
- **MC value function approximation target is the return G_t**

$$w_{t+1} = w_t - \alpha \left(G_t - v_w(s_t) \right) \nabla_w v_w(s_t)$$

- **TD value function approximation target is $r_{t+1} + \gamma \, v_w(s_{t+1})$**

$$w_{t+1} = w_t - \alpha \left(r_{t+1} + \gamma \, v_w(s_{t+1}) - v_w(s_t) \right) \nabla_w v_w(s_t)$$

Monte-Carlo Value Function Approximation

Monte-Carlo Policy Evaluation

- The return G_t is an unbiased, noisy sample of $v_\pi(s)$
- Apply supervised learning to sampled data $\{(s_0, G_0), \ldots, (s_t, G_t)\}$

$$w_{t+1} = w_t - \alpha \left(G_t - v_w(s_t) \right) \nabla_w v_w(s_t)$$

- Monte-Carlo evaluation converges to a local optimum at least (proof out of scope)
- With a linear function approximator, it simplifies to:

$$w_{t+1} = w_t - \alpha \left(G_t - v_w(s_t) \right) \phi(s_t)$$

TD(0) Value Function Approximation

Temporal Difference Policy Evaluation

- The TD target $r_{t+1} + \gamma\, v_w(s_{t+1})$ is a biased sample of $v_\pi(s)$
- Supervised learning can still be applied to sampled data which takes the form $\{(s_0, r_1 + \gamma v_w(s_1)), ..., (s_t, r_{t+1} + \gamma v_w(s_{t+1}))\}$

$$w_{t+1} = w_t - \alpha \left(r_{t+1} + \gamma\, v_w(s_{t+1}) - v_w(s_t) \right) \nabla_w v_w(s_t)$$

- Temporal Difference evaluation converges to a local optimum which may not be the same as the asymptotic MC solution
- MC asymptotic solution unbiased, but TD may converge faster

Human-Level Control with Deep Reinforcement Learning

Control with Value Function Approximation

Approximate action values using a parametric function

$$q_w(s, a) \approx q_\pi(s, a)$$

- Find **w** that minimizes difference between $q_w(s, a)$ and $q_\pi(s, a)$ or $q_*(s, a)$ by stochastic gradient descent:

$$w_{t+1} = w_t - \alpha \left(q_*(s_t, a_t) - q_w(s_t, a_t)\right) \nabla_w q_w(s_t, a_t)$$

- Example: Sample $q_*(s, a)$ as TD target $r_{t+1} + \gamma \max_{a'} q_w(s_{t+1}, a')$ and apply supervised learning while acting on ϵ-greedy policy:

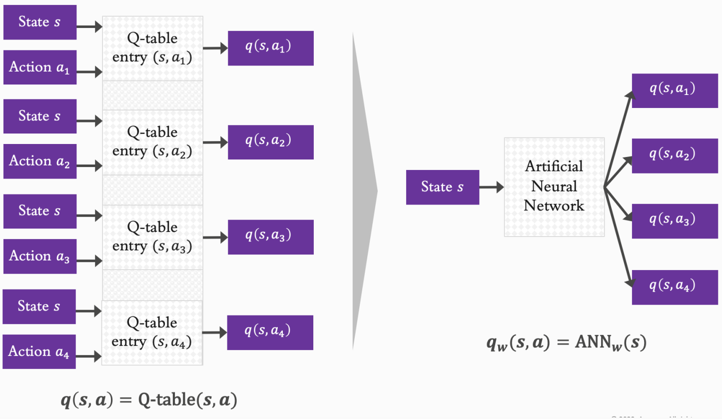
$$w_{t+1} = w_t - \alpha \left(r_{t+1} + \gamma \max_{a'} q_w(s_{t+1}, a') - q_w(s_t, a_t) \right) \nabla_w q_w(s_t, a_t)$$

If $q_w(s, a)$ is a neural network, this is called **Neural Q-Learning**

Control with Neural Q-Learning

Approximate action-values using a Deep Neural Network

- Deep RL: Replace look-up table by an ANN



Practice: Semi-Gradient TD Control

Semi-Gradient TD Control predicts $q(s, a)$ using the same parametric function at every step with no model of the environment

Initialize **w,** $\pi(s)$ and s arbitrarily

Loop forever:

Select and take a from s following $\pi(s)$, observe r_{t+1} and s'

$$w = w - \alpha \left(r_{t+1} + \gamma \max_{a'} q_w(s', a') - q_w(s, a) \right) \nabla_w q_w(s, a)$$

Update π at s given $q_w(s, a)$ by ϵ-greedy soft update

$s = s'$

Convergence of Semi-Gradient TD Control

Semi-Gradient TD Control combines bootstrapping, off-policy learning, and value approximation

- Training data used to parameterize the predictive function is gathered online and thus often non-stationary/not i.i.d.
- The environment itself may continuously evolve, or be too big for exhaustive sampling, and thus may never reach equilibrium
- **Tracking is often preferred to convergence: continually adapt the policy instead of trying to converge to a fixed policy**
- **Theory of control with function approximation still unclear**

Challenges with Semi-Gradient TD Control

Deadly triad: Convergence not guaranteed if we combine these 3 methods together, even with infinite sampling

Bootstrapping

Learn from estimates in neighbor states

Pros: Faster learning and data efficient

Cons: Bias in estimate of target return

Function Approximation

Learn from function generalizing across state space

Pros: Scale to large problems

Cons: Converges iff sampling unbiased estimate of G_t or stationary distribution of states under target policy

Off-Policy

Learn about target policy by following behavior policy

Pros: Learn target behavior from alter streams of experiences

Cons: Bias due to mismatch between expected vs. sampled distribution of states

Improving Quality of RL approximations

Reduce correlation between predicted vs. target values to "makes the training data more i.i.d."

- **Experience Replay:** Keep set of past experiences in memory, find best fitting value function in randomized training batches
- **Add a Target Network:** Use two separate parametric functions, one updated at every step, and one updated periodically which predicts the TD target (frozen within each *temporal period*)
- **Emphatic TD method:** Weight down states observed only in the behavior policy and weight up states observed in target policy (beyond scope)

Practice: Deep Q-Network (DQN)

DQN predicts $q(s, a)$ with two Neural Networks to learn at every step from stable buffers of experiences, with no model of environment

Initialize **w, w$^{\text{target}}$**, $\pi(s)$ and s arbitrarily

Loop forever:

Select and take a from s following $\pi(s)$, observe r_{t+1} and s'

$$w = w - \alpha \left(r_{t+1} + \gamma \max_{a'} q_w^{\text{target}}(s', a') - q_w(s, a) \right) \nabla_w q_w(s, a)$$

Update π at s given $q_w(s, a)$ by ϵ-greedy soft update

Store transition $(s_t, a_t, r_{t+1}, s_{t+1})$ in a Replay Buffer

$s = s'$

Periodically:

Sample mini-batches from the Replay Buffer

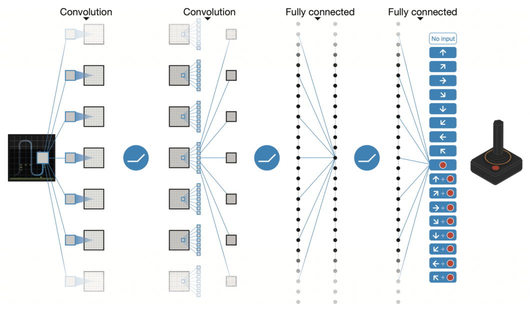
Loop through these mini-batches to further update **w**

Update **w$^{\text{target}}$** $\leftarrow$ **w**

Deep RL Research

Novelty of DQN (Mnih et al., Nature 2015)

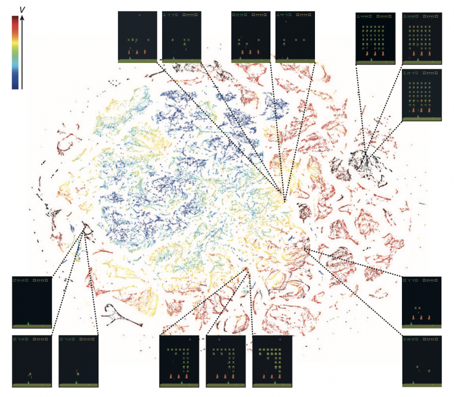
1. **Convolutional Neural Network** creating states from raw pixels
2. **Experience Replay buffer** storing set of past experiences $e_i = (s_i, a_i, r_{i+1}, s_{i+1})$ from which minibatches are randomly sampled
3. **Target network** updated periodically (frozen between updates)



Deep RL Research

Perceptually similar states are mapped to nearby points in the high-dimentional CNN embedding space

- 2D t-SNE representations in last CNN layer assigned to Space Invaders states

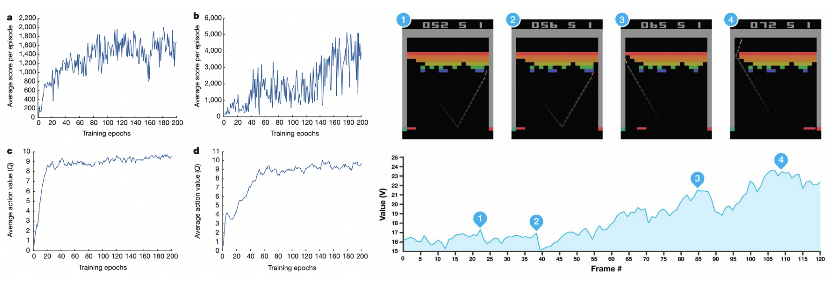


Mnih et al., Nature 2015

Deep RL Research

DQN can train a large ANN with stochastic gradient descent in a stable manner to identify winning actions

- Temporal evolution of average score-per-episode and predicted Q-values



Mnih et al., Nature 2015

Deep RL Research

Disabling DQN's replay memory and/or separate target Q-network has a detrimental effect on performance

- Effects of Experience Replay and Separate Target Q-Network on performance

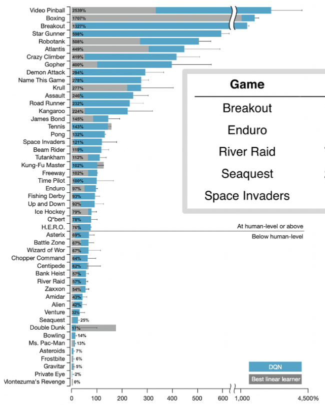
Game	With replay, with target Q	With replay, without target Q	Without replay, with target Q	Without replay, without target Q
Breakout	316.8	240.7	10.2	3.2
Enduro	1006.3	831.4	141.9	29.1
River Raid	7446.6	4102.8	2867.7	1453.0
Seaquest	2894.4	822.6	1003.0	275.8
Space Invaders	1088.9	826.3	373.2	302.0

Mnih et al., Nature 2015

Deep RL Research

DQN outperforms all linear methods and achieve level comparable to that of professional human players

- Comparison of DQN performance vs. best RL from 2015 on 49 Atari video games



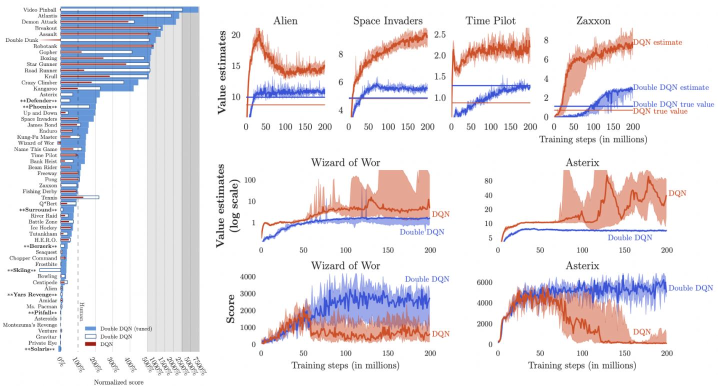
Game	DQN	Linear
Breakout	316.8	3.00
Enduro	1006.3	62.0
River Raid	7446.6	2346.9
Seaquest	2894.4	656.9
Space Invaders	1088.9	301.3

Mnih et al., 2015

Deep RL Research

DDQN outperforms DQN by reducing the overoptimism due to value estimation errors

- Comparison of DDQN performance vs. DQN on 57 Atari video games



 van Hasselt, Silver, 2015

Deep RL Research

Deep RL is a fertile research area

- **Performance has improved dramatically in past few years:**
  - **Deep Q-Network (2014)**
  - **Deepmind AlphaGo (2016)**
  - **Deepmind AlphaZero (2018)**
  - **OpenAI ChatGPT (2022)**
- **Some key open questions:** (lectures 6-9)
  - How best to build objective measures to optimize function parameters?
  - Can we directly parameterize and optimize a policy function?
    Would such direct policy optimization benefit from also learning values?
  - Can we learn a model of the environment? How best to use a model?
  - How best to build the agent state? (including what it stores in memory)
  - How best to improve data sampling efficiency?

DQN learns to play Atari

Deep Q-network training on video game Seaquest

(Source: Sprague N., 2015)

Thank you!