DS-GA 3001 007 | Lecture 7

Reinforcement Learning

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DS-GA 3001 RL Curriculum

Reinforcement Learning:

- Introduction to Reinforcement Learning
- Multi-armed Bandit
- Dynamic Programming on Markov Decision Process
- Model-free Reinforcement Learning
- Value Function Approximation (Deep RL)
- **Policy Function Approximation (Actor-Critic)**
- Planning from a Model of the Environment
- Examples of Industrial Applications
- Advanced Topics and Development Platforms

Reinforcement Learning

Lecture 5: Value Function Approximation

- Categories of Functions in Reinforcement Learning
- Approximation of State-Update and Value Functions
- Deep Reinforcement Learning

Today: Policy Function Approximation

- **Policy Gradient Reinforcement Learning**
- **Advanced Sampling of Policy Gradient**
- **Reinforcement Learning in Continuous Action Space**

RL Functions and Approximations

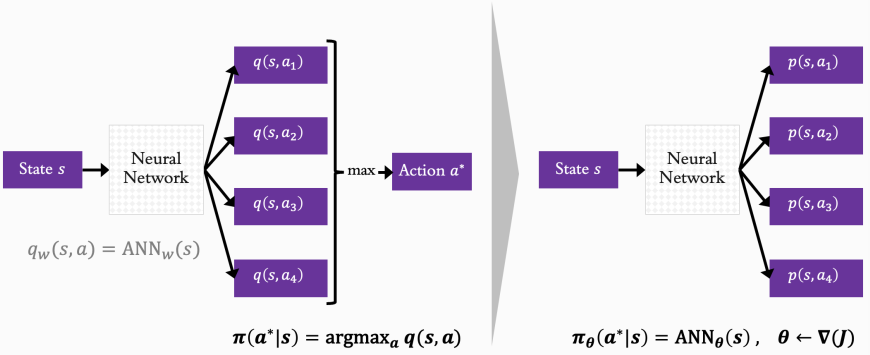
- **All key components of an RL agent are functions**
  - **State update functions** map observations to states
  - **Value functions** map states to values
  - **Policy functions** map states to actions
  - **Models** map states and actions to next states and rewards
- **Functions can be parameterized and approximated** by linear or non-linear gradient methods e.g., Deep Learning
- **A parameterized policy function is called an ”actor” function**. Policies are action-selection functions which can be derived determistically from a value function (example: ϵ-greedy soft updates), but more generally they can be *parameterized*
- **Challenge:** Supervised learning assumptions are often violated

Policy Gradient Reinforcement Learning

Policy-based RL

Predict optimal action using a Deep Neural Network

- Replace ϵ-greedy function by an ANN model



Policy-based RL

Optimize a Parametric Policy Function

$$\pi_\theta(a|s) \approx p(a|s, \theta)$$

- The function can be a linear combination of features, often with a soft-max to define action-probabilities normalizing to 1:

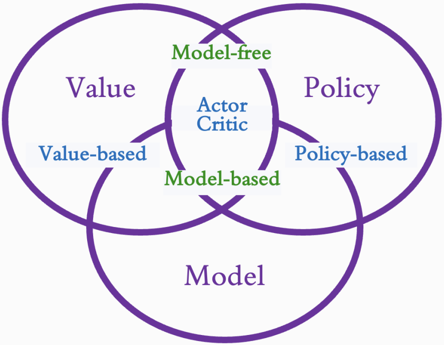
$$\pi_\theta(a|s) = \frac{e^{\theta^T \phi(s,a)}}{\sum_k e^{\theta^T \phi(s,a_k)}}$$

- Parameters θ can be updated incrementally by optimizing any objective measure of performance $J(\theta)$:

$$\theta \sim \arg\max_\theta J(\theta)$$

- Gradient ascent converges to global optimum if $J(\theta)$ is known

Policy-based RL



Categories of RL agents:

- **Model-based**: Use a model to learn policy and/or value function (lecture 3, 8)
- **Model-free**: Learn policy and/or value function without model (lecture 4-5)
- **Value-based**: Learn value function, not policy function (lectures 4-5)
- **Policy-based**: Learn policy function, not value function (today)
- **Actor-Critic**: Learn policy and value functions (today)

Model-, Value-, or Policy-based RL ?

- **Model-based RL:**
  - ✓ Learns 'all there is to know' from the data
  - ✓ Very well understood method (supervised learning)
  - × Objective captures irrelevant information
  - × May focus compute/capacity on irrelevant details
  - × Deriving a policy from a model (planning) is non-trivial
- **Value-based RL:**
  - ✓ Focus compute/capacity on how actions affect reward
  - ✓ Relatively well-understood (similar to regression)
  - × May still focus compute/capacity on irrelevant details
- **Policy-based RL:**
  - ✓ Direct search of optimal policy = true objective of RL
  - × Ignores all other learnable knowledge

Pros & Cons of Policy-based RL

- **Strengths:**
  - ✓ Focus on true objective of RL. Sometimes policies are simple while values and models are complex
  - ✓ Can learn high-dimensional or continuous action policies
  - ✓ Can learn stochastic (= probabilistic) policies
  - ✓ Can learn appropriate levels of exploration autonomously (no need to manually define an ϵ-exploration schedule)
- **Limitations:**
  - × Ignores all other learnable knowledge so may not efficiently use the available data
  - × Easily trapped in local optima (actor is what creates data)
  - × May not generalize well when environment changes

Policy Optimization using Gradient

Define Policy's Objective Measure of Quality

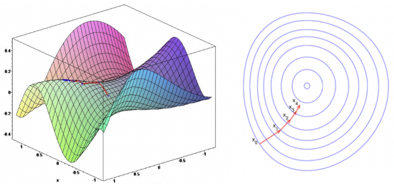
- **Ex 1 (Episodic Case):** Maximize expected return in s_0:

$$J_0(\theta) = \mathop{\mathbb{E}}_{\pi_\theta}(G_t | s_0) = v_{\pi_\theta}(s_0)$$

- **Ex 2 (Continuous Case):** Maximize average return across states:

$$J_{avg}(\theta) = \mathop{\mathbb{E}}_{\pi_\theta}(G_t | s) = \sum_s d_{\pi_\theta}(s)\, v_{\pi_\theta}(s)$$

Update policy parameters by stochastic gradient ascent



Policy Optimization using Gradient

Update policy parameters by stochastic gradient ascent

- To find a maximum of $J(\theta)$, define the gradient of $J(\theta)$ and move θ in the direction of positive gradient at every step:

$$\nabla_\theta J(\theta) = \begin{bmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{bmatrix}$$

$$\theta_{t+1} = \theta_t + \alpha \nabla_\theta J(\theta)$$

- **Some methods do not use gradient (out of scope)**:
  - Hill climbing
  - Genetic algorithms

Policy Optimization using Gradient

How can the agent learn

$$\nabla_\theta J(\theta) = \nabla_\theta \, \mathbb{E}_{\pi_\theta}(G_t|s)$$

?

Sample what? Compute what?

Policy Gradient Theorem

Theorem's Proof

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \nabla_\theta \mathop{\mathbb{E}}_{\pi_\theta}(G_t|s) \\
&= \nabla_\theta \sum_s \mu(s) \sum_a \pi_\theta(a|s)\, G_t \\
&= \sum_s \mu(s) \sum_a \nabla_\theta\, \pi_\theta(a|s)\, G_t \\
&= \sum_s \mu(s) \sum_a \pi_\theta(a|s) \frac{\nabla_\theta\, \pi_\theta(a|s)}{\pi_\theta(a|s)}\, G_t \\
&= \mathop{\mathbb{E}}_{\pi_\theta}\left(G_t \frac{\nabla_\theta\, \pi_\theta(a|s)}{\pi_\theta(a|s)}\right) = \mathop{\mathbb{E}}_{\pi_\theta}\left(G_t\, \nabla_\theta\, \log \pi_\theta(a|s)\right)
\end{aligned}
$$

- **In plain english:** Gradient of the expected value of G_t under π_θ equals the expected value of G_t weighted by gradient of $\log \pi_\theta$

Policy Gradient Theorem

$\forall$ differentiable policy π_θ, the gradient of $J = \mathbb{E}_{\pi_\theta}(G_t|s)$ is:

$$\nabla_\theta \mathop{\mathbb{E}}_{\pi_\theta}(G_t|s) = \mathop{\mathbb{E}}_{\pi_\theta}\left(G_t \frac{\nabla_\theta \pi_\theta(a|s)}{\pi_\theta(a|s)}\right) = \mathop{\mathbb{E}}_{\pi_\theta}(G_t \nabla_\theta \log \pi_\theta(a|s))$$

Theorem's Implications

- The gradient of $J(\theta)$ can be sampled because it is equal to the expected value of some known quantities (G_t, π_θ, and $\nabla_\theta \pi_\theta$)
- Gradient updates of π_θ do not involve derivatives of the state distribution thus are agnostic to detailed dynamics of the MDP

Practice: MC Policy Gradient (REINFORCE)

REINFORCE update $\pi_\theta(a|s)$ using the policy gradient computed over complete episodes, with no model of the environment

Initialize θ arbitrarily

Loop forever:

Initialize s_0

Experience an episode $(s_0, a_0, r_1, s_1, a_1, r_2, ..., r_T)$ following π_θ:

Loop for each step t of episode:

$$G = r_{t+1} + \gamma\, r_{t+2} + ... + \gamma^T\, r_{t+1+T}$$

$$\theta = \theta + \alpha\, G\, \nabla_\theta \log(\pi_\theta(a|s))$$

Advanced Sampling of Policy Gradient

Advanced Sampling of the Gradient $\nabla_\theta J(\theta)$

- **Variance can be reduced by adding a baseline to the MC target:**

$$\theta_{t+1} = \theta_t + \alpha\,(G_t - v_\pi(s_t))\,\nabla_\theta \log \pi_\theta(a|s_t)$$

- **It is called the *advantage* of following π_θ relative to baseline:**

$$\theta_{t+1} = \theta_t + \alpha\,(q_\pi(s_t, a) - v_\pi(s_t))\,\nabla_\theta \log \pi_\theta(a|s_t)$$

- **A TD target can be used to learn at every step:**

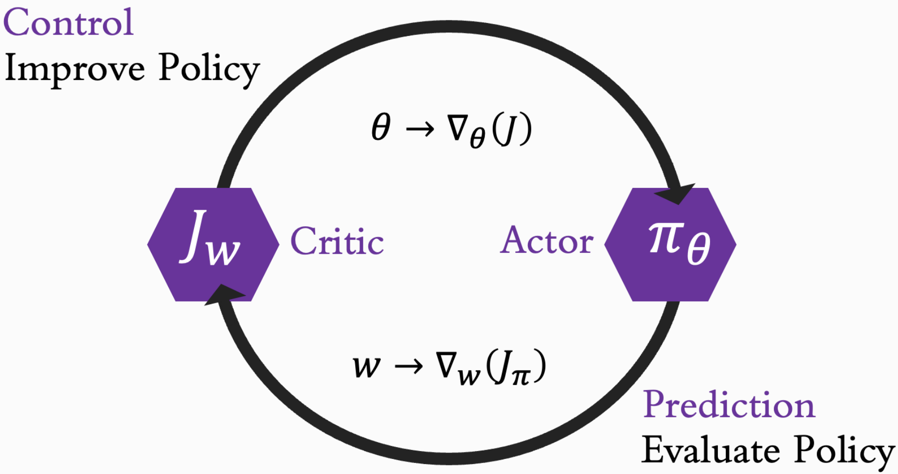
$$\theta_{t+1} = \theta_t + \alpha\,(r_{t+1} + \gamma\, v_\pi(s_{t+1}) - v_\pi(s_t))\,\nabla_\theta \log \pi_\theta(a|s_t)$$

- **Value function approximation refines (*critics*) the gradient:**

$$\theta_{t+1} = \theta_t + \alpha\,(r_{t+1} + \gamma\, v_w(s_{t+1}) - v_w(s_t))\,\nabla_\theta \log \pi_\theta(a|s_t)$$

Generalized Policy Iteration

All RL methods are Generalized Policy Iteration methods



Practice: A3C Actor-Critic

Asynchronous Advantage Actor-Critic (A3C) updates π_θ using the policy gradient computed from v_w at every step, in any state s

Initialize w, θ, and s, arbitrarily

Loop forever:

Take a from s following $\pi_\theta(a|s)$, observe r_{t+1} and s'

$\delta = r_{t+1} + \gamma\, v_w(s_{t+1}) - v_w(s_t)$ [TD-error = Advantage]

$w_{t+1} = w_t - \alpha_1\, \delta\, \nabla_w v_w(s_t)$ [Update of Critic]

$\theta = \theta + \alpha_2\, \delta\, \nabla_\theta \log \pi_\theta(a|s_t)$ [Update of Actor]

$s = s'$

Bias-variance tradeoff in A3C

Update policy gradient from multi-step TD error

- The full return $G_t = (r_{t+1} + \gamma\, r_{t+2} + \ldots)$ has high variance
- The TD target $r_{t+1} + \gamma\, v_w(s_{t+1})$ has high bias
- A useful middle ground is to define TD error with n-step return:

$$G_{t:t+n} = r_{t+1} + \gamma r_{t+2} + \ldots + \gamma^{n-1} r_{t+n} + \gamma^n v_w(s_{t+n})$$

- Or to define TD error with λ-returns as in TD(λ):

$$G_t^\lambda = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1}\, G_{t:t+n}$$

- **BUT both require more compute/memory (can be prohibitively too slow), and must wait *n* steps before learning update occurs**

Regularisation of Policy Function

A biased policy samples biased data

- The policy gradient objective only considers improvement under observed/sampled data
- But the policy is what drives the sampling of data, so the agent gets easily trapped in local optima, which perpetuate biases
- Entropy needs be increased (encourage exploration) without causing breakage due to widly different policies (instability)
- Many variants from original A3C method have been developed to address the bias-variance tradeoff of actor-critic RL
- For example to prevent instability and perpetuating biases, the policy can be regularised to not change too much

Regularisation of Policy Function

Trust Region Policy Optimization (TRPO)

- Prevent instability by regularizing $J(\theta)$ to limit the difference between subsequent policies (= limit speed of exploration)
- The difference between two probability distributions/densities is the Kullback-Leibler divergence D_{KL}:

$$D_{KL}(\pi_{old}||\pi_\theta) = \sum_a \pi_{old}(a|s) \log \frac{\pi_\theta(a|s)}{\pi_{old}(a|s)}$$

- TRPO defines a "trust-region" by maximizing this objective:

$$\nabla_\theta \left[\mathop{\mathbb{E}}_{\pi_{\text{old}}} \left(\delta \frac{\pi_\theta(a|s)}{\pi_{\text{old}}(a|s)} \right) - \eta\, D_{KL}(\pi_\theta||\pi_{\text{old}}) \right]$$

- TRPO is especially useful for KT from good starting policies

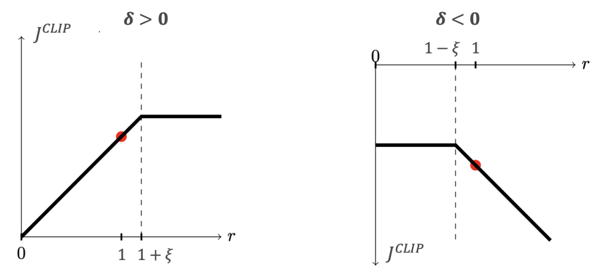
 Schulman et al. (2015)

Regularisation of Policy Function

Proximal Policy Optimization (PPO)

- PPO defines a trust region by regularizing the objective with a clipped probability ratio (similar to TRPO but simpler)

$$\nabla_\theta \left[\mathop{\mathbb{E}}_{\pi_{\text{old}}} \left(\delta \ \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\text{old}}(a|s)}, 1-\xi, 1+\xi \right) \right) \right]$$



 Schulman et al. (2017)

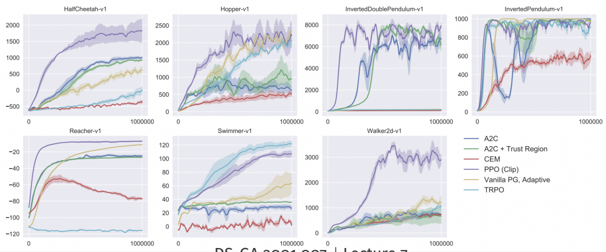
Regularisation of Policy Function

Proximal Policy Optimization (PPO)

- PPO defines trust region by regularizing objective with clipped probability ratio (same results as TRPO but simpler algorithm)

$$\nabla_\theta \left[\mathop{\mathbb{E}}_{\pi_{\text{old}}} \left(\delta \ \text{clip}\left(\frac{\pi_\theta(a|s)}{\pi_{\text{old}}(a|s)}, 1-\xi, 1+\xi \right) \right) \right]$$

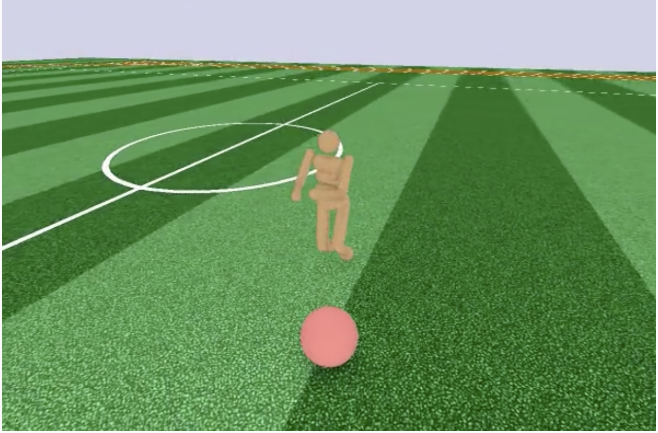
- Benchmark of PPO performance on MuJoCo Gym environments



Schulman et al. (2017)

Example: Moving a Robot with PPO

Proximal Policy Optimization (PPO)



For more details: https://openai.com/blog/openai-baselines-ppo

Example: Moving a Robot with PPO

Proximal Policy Optimization (PPO)

(Source: DeepMind (2017))

Reinforcement Learning in Continuous Action Space

Policy Gradient for Continous Action Space

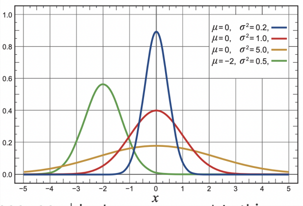
Policy Gradient easily extends to continous actions

- Instead of learning probabilities for discrete actions, RL can learn a statistics of continuous action-probability densities
- For example, an action can be chosen from a Gaussian density:

$$\pi_\theta(a|s) \sim \mathcal{N}\left(\mu_\theta(s), \sigma_\theta^2\right)$$

$$\forall\, \sigma^\dagger : \nabla_\theta J(\theta) = \mathbb{E}\left(\delta\, \nabla_\theta \log \pi_\theta(a|s_t)\right) \approx \delta\, \frac{a - \mu_\theta(s)}{\sigma^2}\, \nabla \mu_\theta(s)$$

- Sampling actions from a probability density guarantees exploration



† In this example σ is predefined

Practice: CACLA algorithms

Continuous Actor-Critic Learning Automaton (CACLA) samples actions from a parameterized probability density $\pi_\theta \sim \mathcal{N}\left(\mu_\theta, \sigma_\theta^2\right)$, udpated from the gradient of v_w at every step, in any state s

Initialize w, θ, and s, arbitrarily

Loop forever:

Take a from s following $\mathcal{N}\left(\mu_\theta(s), \sigma_\theta^2\right)$, observe r_{t+1} and s'

$\delta = r_{t+1} + \gamma\, v_w(s_{t+1}) - v_w(s_t)$

$w_{t+1} = w_t - \alpha_1\, \delta\, \nabla_w v_w(s_t)$

If* $\delta > 0$: $\theta = \theta + \alpha_2\, \delta\, \nabla_\theta \log \pi_\theta(a|s_t)$

$s = s'$

* The original CACLA algorithm (2007) updated actor iff sampled a_t increases value

Practice: DDPG algorithms

Deep Deterministic Policy Gradient (DDPG) generalizes CACLA and DQN by sampling actions from a probability density $\pi_\theta \sim \mathcal{N}\left(\mu_\theta, \sigma^2\right)$ udpated directly from the gradient of q_w learned by deep learning

Initialize w, θ, and s, arbitrarily

Loop forever:

Take a from s following $\mathcal{N}(\mu_\theta(s), \sigma^2)$, observe r_{t+1} and s'

$w_{t+1} = w_t - \alpha_1 \left(r_{t+1} + \gamma\, q_w\left(s_{t+1}, \mu_\theta\left(s_{t+1}\right)\right) - q_w\left(s_t, a\right)\right) \nabla_w q_w(s_t, a)$

$\theta = \theta + \alpha_2\, \nabla_\theta q_t\left(s_t, \mu_\theta(s_t)\right)$

Store past transitions in a Replay Buffer

$s = s'$

Periodically:

Sample mini-batches from the Experience Replay Buffer

Loop through these mini-batches to further update w and θ

* Lillicrap, Silver et al. (2016)

Thank you!